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PENETRATION OF A COPPER ROD INTO A SANDY TARGET

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This paper presents the results of experimental and theoretical studies of high-velocity penetration of cylindrical copper rods into sand. The hydrodynamic Alekseevskii–Tate theory is modified to determine the penetration depth and wear velocity of the material of the rod penetrating into soil target in the plastic and hydrodynamic stages of penetration. The case where the target material is significantly less strong than the rod (*impactor*) *material is considered.*

Key words: *high-velocity penetration, plastic deformation, hydrodynamic transition, plastic wave, yield point.*

Introduction. The high-velocity penetration of long rods (impactors) into dense media includes two stages of deformation [1–3]: 1) plastic stage, where the decrease in the length of the solid part of the impactor occurs at a rate equal to the velocity of the longitudinal plastic wave C_p; 2) hydrodynamic stage, where the wear velocity of the impactor $V - U$ exceeds the velocity of the longitudinal wave of plastic deformation (V is the velocity of the undeformed tail part of the impactor and U is the penetration velocity).

The hydrodynamic stage of high-velocity penetration is adequately described by a modified hydrodynamic Alekseevskii–Tate theory (MHT) [4–8] based on the results of experimental studies of penetration of metal rods into high-strength (mainly metal) targets. However, high-velocity penetration of metal rods into porous soft soil media has been studied insufficiently (see, for example, [9]).

One of the key parameters characterizing the high-velocity interaction of an impactor with a target in the hydrodynamic stage of penetration is the coefficient of relative penetration $K = h/\Delta L$ [h is the current penetration depth and ΔL is the shortening (wear) of the impactor]. The coefficient K allows one to predict the total wear depth of the impactor in the hydrodynamic stage in the case of stationary penetration. Stationarity is provided by a great (not less than sevenfold) elongation of the impactor (see, for example, [7]). According to the Alekseevskii–Tate model, the quantity K depends on the penetration velocity V_0. As V_0 increases, the value of K decreases, approaching the asymptotic value given by the well-known Lavrent'ev formula [10] for the case of an ideal incompressible fluid: $K = \sqrt{\rho_p/\rho_t}$.

The objectives of the present work was to experimentally determine the critical velocity V_* at which plastic deformation begins, the velocities of hydrodynamic transition V_{ht}, and the velocities of a longitudinal plastic wave in a copper rod during its penetration into sand C_p, and to improve the MHT to describe high-velocity penetration taking into account two stages of deformation of the impactor in the case where the target material is significantly less strong than the impactor material.

1. Results of Experiments. The cylindrical copper impactors fired from ballistic facilities into soil had diameter $d = 1$ cm, length $L_0 = 8.22$ cm, and mass $m = 57.5$ g. The impactors were made of M1 copper alloy with a conventional yield point in compression $\sigma_{0.2} \approx 290$ MPa [11] and density $\rho_p = 8.9$ g/cm^3. Experimental data were obtained by x-ray photography of the penetration process. The soil target was a container filled with sand which had the following dimensions: width (8–10)d, height not less 30d, and length approximately equal to 50d. Fine-grained sand of density $\rho_t = 1.65$–1.75 g/cm^3 with a moister content $W = 5$–10% was used. From the x-ray photograph

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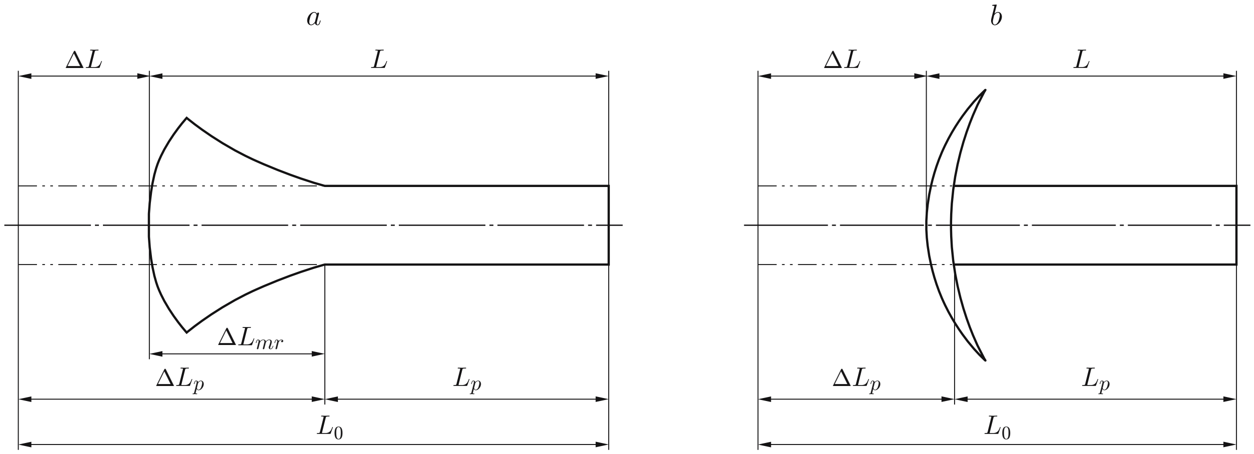

Fig. 1. Diagram of impactor deformation during high-velocity penetration into sand: (a) plastic stage of penetration; (b) hydrodynamic stage of penetration.

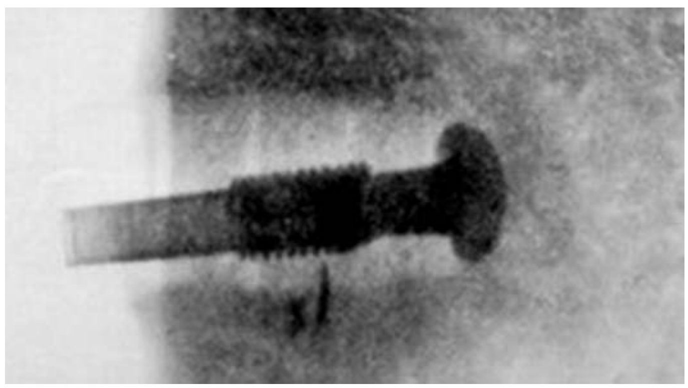

Fig. 2. X-ray photograph of penetration of a copper rod ($V_0 \approx 1040$ m/sec).

obtained at time t we determined the current penetration depth h, the current length of the impactor L, and the length of the tail undeformed part of the impactor L_p. These parameters were used to determine the shortening of the impactor $\Delta L = L_0 - L$ and the length of its deformed part $\Delta L_p = L_0 - L_p$ (Fig. 1). The value of L_p was reckoned from the rear end of the impactor to the section at which the impactor radius exceeded the initial value by 5%. This measurement method can lead to systematic overestimation of values of L_p and, hence, underestimation of values of ΔL_p. It was assumed that, in the hydrodynamic stage of penetration, $L_p = L$ ($\Delta L_p = \Delta L$) due to the insignificant (compared to the length of the rod) thickness of the liquid phase of the impactor material on the contact boundary.

Figure 2 shows a typical x-ray photograph of impactor penetration in the plastic stage of deformation. On the rod one can see the thread onto which a spade cover was screwed to accelerate and hold the rod in the barrel. The critical velocity is determined for $V = U = V_*$ from the equality [4, 5] of the pressures at the rod tip on the side of the soil and on the side of the rod:

$$\rho_p(V - U)^2/2 + Y_p = C_0\rho_t U^2/2 + R_t. \tag{1}$$

Here Y_p is the dynamic yield point of the impactor material above which it enters a plastic or liquid state [4] (Y_p depends on pressure), R_t is the dynamic hardness of the target material [4–7], which is negligibly small compared to Y_p. Because in the proposed model, the soil target has a significant compressibility, in contrast to the ideal incompressible fluid model, we take into account the pressure deceleration coefficient C_0, whose value exceeds unity. The critical velocity is equal to

$$V_*^2 = 2(Y_* - R_t)/(\rho_t C_0)$$

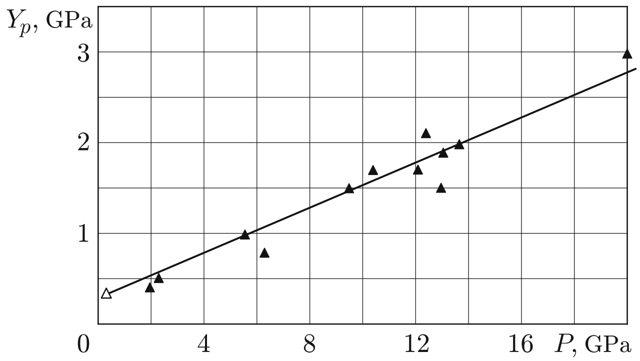

Fig. 3. Yield point of copper versus pressure: points are experimental data for M1 copper alloy and [13], solid curve shows the results of calculations using formula (3).

(Y_* is the yield point of the impactor material at the critical penetration velocity). The deceleration pressure coefficient can be determined by various methods, for example, approximately, by taking into account the ratio of the density of the unperturbed layer of the target to the density in the shock-compressed layer [7]. It is also known hat the pressure coefficient C_0 is close to the drag coefficient C_x of the flat end of a cylinder. It is found that for the sand used in the experiments, the drag coefficient of the flat end of a cylinder at velocities of 100–1000 m/sec is $C_x = 1.50 \pm 0.09$ (± 0.09 is the standard deviation) [12]. In the calculations, instead of the value of C_0 we shall use the experimentally obtained value of C_x for the flat end. In this case, there is no need to take into account the value of R_t since it is already included in the value of the overall coefficient. By increasing the penetration velocity in the experiments, we obtained the value $V_* \approx 500$ m/sec and, hence (for $R_t \ll Y_*$) $Y_* \approx 320$ MPa. This value of Y_* is in good agreement with the value of the yield point in compression [11] for the obtained deformation velocity approximately equal to V_*/L_0. It is assumed that the hydrodynamic mode begins when the impactor velocity exceeds the velocity of hydrodynamic transition V_{ht}, which is given by the relation (1) for $V - U = C_p$ [1]. If the yield point is constant, the velocity V_{ht} is determined from the formula [1]

$$V_{ht} = C_p + \Big[V_*^2 + \frac{1}{C_0}\Big(\frac{C_p}{\mu}\Big)^2\Big]^{1/2},$$

where $\mu = (\rho_t/\rho_p)^{1/2}$.

Tables 1 and 2 give the results of measurements of ΔL and ΔL_p. The velocity of the longitudinal plastic wave C_p was determined by x-ray photographs using the formula $C_p = \Delta L_p/t$. From the data of 10 experiments, we obtained the value $C_p = (437 \pm 9)$. The experiments did not reveal a dependence of C_p on the impact velocity in the range of velocities considered. The obtained value is in good agreement with the velocity of the longitudinal plastic wave given by the formula $C_p = \sqrt{E_y/\rho_p}$. Here E_y is the plastic hardening modulus (in [1], for copper, $E_y = 1.86$ GPa, i.e., $C_p = 456$ m/sec). Tables 1 and 2 give values of the deformation coefficient $K_p = h/\Delta L_p$ (in the plastic stage) and the relative penetration coefficient $K = h/\Delta L$ for both stages.

2. Modified MHT. According to the approximate theory, the system of equations describing the penetration of an impactor, along with Eq. (1), has the form

$$\frac{dh}{dt} = U, \qquad \rho_p L_p \frac{dV}{dt} = -Y_p(V),$$

$$\frac{dL_p}{dt} = -C_p \quad \text{at} \quad V_* \leqslant V < V_{ht}, \qquad L_p = L \quad \text{at} \quad V \geqslant V_{ht}, \tag{2}$$

$$\frac{dL}{dt} = -(V - U).$$

Unlike the classical modified hydrodynamic theory, Eqs. (2) take into account the plastic stage of penetration and the dependence of the yield point of the impactor material on the penetration velocity (during motion of the impactor, the velocity of its solid part decreases) i.e., the hardening of the rod material is taken into account. Thus, system (2) also describes the plastic stage of deformation of the rod. To obtain the dependence $Y_p(V)$, we

TABLE 1

Penetration of a Copper Impactor into Sand in the Plastic Stage

V_0, m/sec	Experimental data							Calculation using IMHT				
	h, mm	t, μsec	ΔL, mm	ΔL_p, mm	K_p	K	C_p, m/sec	t, μsec	ΔL, mm	ΔL_p, mm	K_p	K
703	27.5	44.2	5.5	20	1.37	5.00	452	48.1	5.7	21.0	1.31	4.82
853	70.0	96.8	19.0	42	1.67	3.68	434	109.4	19.3	47.8	1.46	3.63
879	65.0	93.8	20.0	41	1.58	3.25	437	98.9	18.7	43.2	1.51	3.48
965	62.0	81.9	19.5	35	1.77	3.18	427	87.4	19.7	38.2	1.62	3.15
1080	57.0	67.8	19.0	30	1.90	3.00	442	72.9	19.3	31.8	1.79	2.95
1125	53.0	61.0	18.0	27	1.89	2.94	443	65.3	18.9	28.5	1.86	2.80
1214	125.0	136.0	48.0	58	2.15	2.60	426	149.4	45.3	65.3	1.91	2.76
1218	120.0	141.0	45.0	60	2.00	2.67	426	142.5	43.7	62.2	1.93	2.75
1225	90.0	96.0	32.0	42	2.14	2.81	438	104.4	33.2	45.6	1.97	2.71
1330	102.0	104.0	38.5	46	2.21	2.65	442	110.3	39.0	48.2	2.12	2.62

TABLE 2

Penetration of a Copper Impactor in Sand in the Hydrodynamic Stage

V_0, m/sec	Experimental data				Calculation using IMHT		
	h, mm	t, μsec	ΔL, mm	K	t, μsec	ΔL, mm	K
1544	105	104.0	44	2.39	98.8	42.3	2.48
1590	155	153.0	62	2.50	145.3	62.4	2.48
1636	65	58.4	28	2.32	57.0	26.7	2.43
1639	63	60.0	25	2.52	55.2	25.9	2.43
1650	105	97.0	42	2.50	92.8	43.0	2.44
2000	125	102.0	52	2.40	92.5	53.2	2.35
2006	135	110.0	58	2.33	100.2	57.4	2.35

used experimental results [13] on the yield point for copper. Using the condition for one-dimensional deformation $\sigma_x = P + 2Y_p/3$, it is possible to obtain the dependence $Y_p(P)$. Figure 3 gives the experimental dependence $Y_p(P)$ and its linear approximation. In the experiments performed, the pressure arising during impactor penetration into the sand (ignoring the impact nonstationary stage) did not exceed 10 GPa. At $Y_p \geqslant Y_*$ and loading up to small axial stresses ($\sigma_x \leqslant 20$–25 GPa), where the temperature effect is insignificant, the dependence $Y_p(P)$ can be considered linear [13, 14]:

$$Y_p = Y_0 + kP \tag{3}$$

($k = 0.125$ and $Y_0 = 280$ MPa). The values of Y_0 and k are determined by the least-squares method. The light point in Fig. 3 corresponds to the value $Y_p = Y_* = 320$ MPa at which the plastic stage of deformation begins. According to (3), the pressure at this point is $P = 320$ MPa. In relation (3), the pressure at the beginning of plastic deformation of the rod material at a given penetration velocity can be calculated approximately using the formula

$$P = C_0 \rho_t V^2/2. \tag{4}$$

The value $P = 320$ MPa corresponds to $V = 500$ m/sec, i.e., the value of V_* obtained in the experiment. Relations (3) and (4) define the dependence $Y_p(V)$. During penetration accompanied by deformation of the impactor, the velocity of its solid part V can vary from V_0 to V_*. The yield point of the impactor material also changes. Thus, system (1)–(4) completely describes impactor penetration in both the plastic and hydrodynamic stages. In view of the hardening of the impactor material $Y_p(V)$, the velocity of hydrodynamic transition is determined from the equation

$$V_{ht} = \frac{C_p}{1-k} + \Big[V_*^2 + \frac{C_p^2}{(1-k)^2}\Big(k + \frac{1-k}{C_0\mu^2}\Big)\Big]^{1/2}.$$

During penetration of the copper impactor into sand, the velocity of hydrodynamic transition with allowance for material hardening is $V_{ht} = 1521$ m/sec.

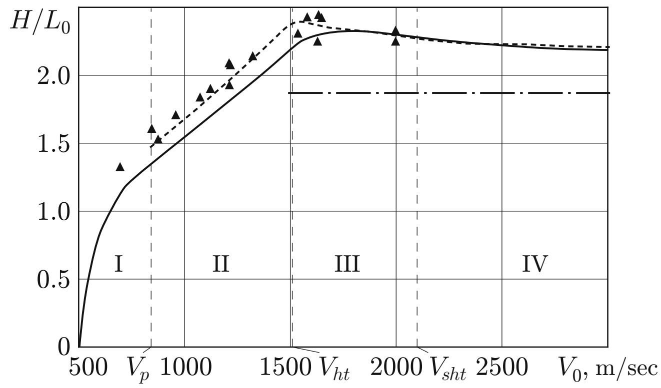

Fig. 4. Deformation depth versus impactor velocity for $V_* \leqslant V_0 < V_p$ (I), $V_p \leqslant V_0 < V_{ht}$ (II), $V_{ht} \leqslant V_0 < V_{sht}$ (III), and $V_0 \geqslant V_{sht}$ (IV): solid curve corresponds to the calculation using the IMHT, dashed curves to the estimate obtained by formula (8) with Eqs. (6) and (7) taken into account, and the dot-and-dashed curve corresponds to the estimate using Lavrent'ev's formula (5); points are experimental data.

3. Calculated Parameters of Penetration a Copper Rod. Results of calculations using the improved modified hydrodynamic theory (IMHT) are also given in Tables 1 and 2. The results were compared with experimental data for fixed values of V_0 and h. Generally, the difference between the calculated values of t and ΔL (ΔL_p) does not exceed 10%, i.e., it is within the measurement error. Thus, the IMHT theory adequately describes the penetration process at both stages of deformation of the impactor material. The theory presented above was used to determine a number of qualitative and quantitative characteristics of the penetration process. As the initial parameters of the impactor and target in the calculations, we used the same data as in the experiments.

Generally, the impactor penetration into a low-strength target can include the hydrodynamic, plastic, and solid stages. By the solid stage is meant the motion of the impactor after the end of its rigid deformation with deformed tip. In the hydrodynamic stage, the shortening of the impactor is accompanied by wear, i.e., a reduction of its mass, whereas in the plastic stage (in the case of a copper impactor) its plastic compression occurs without mass change.

Below, we determine the penetration depth H corresponding to the end of deformation, i.e., conversion of the rod to a solid residue or a plastically deformed residue. In the hydrodynamic stage, the quantity H is the total wear depth. The dependence of the final penetration depth on the initial impactor velocity and five characteristic regions of this dependence are given in [7]. This dependence is a generalization of numerous experiments, mainly for the case of penetration into strong targets. In the case of penetration of a metal impactor into sand, calculations using the IMHT made it possible to distinguish four characteristic ranges of penetration velocities (the range of velocities corresponding to solid-state motion is not considered in the present paper): $V_* \leqslant V_0 < V_p$ (region I), $V_p \leqslant V_0 < V_{ht}$ (region II), $V_{ht} \leqslant V_0 < V_{sht}$ (region III), and $V_0 \geqslant V_{sht}$ (region IV).

In region I, the plastic stage of penetration is always followed by the solid stage, and in region II, only the plastic stage of penetration takes place. Impactor deformation in region II results in a plastically compressed copper impactor which has the same mass as the initial one. In region III, the hydrodynamic stage of wear of the impactor is followed by the plastic stage of deformation, in which final plastic compression of the impactor occurs. At a penetration velocity $V_0 \geqslant V_{sht}$, complete hydrodynamic wear of the impactor occurs. In the calculations, the values $V_p = 1.7V_* = 850$ m/sec and $V_{sht} = 2100$ m/sec were obtained. In the case $V_0 = V_{sht}$, the impactor is decelerated to a velocity equal to the velocity of hydrodynamic transition V_{ht} and completely wears away. In the calculations, it was assumed that the copper impactor completely worn away when the solid residue was less than 3% of the initial length. Figure 4 gives calculated and experimental dependences of the depth corresponding to the end of impactor deformation on penetration velocity in all four regions. At $V_0 \geqslant V_{sht}$, the quantity H corresponds to the complete wear of the impactor and is almost equal to the complete penetration depth, The maximum value of the depth corresponding to the end of deformation takes place in region III at $V_0 \approx 1750$ m/sec, i.e., at a value of V_0 equal to the average value of the velocity in region III. Figure 4 shows the values of the penetration depth

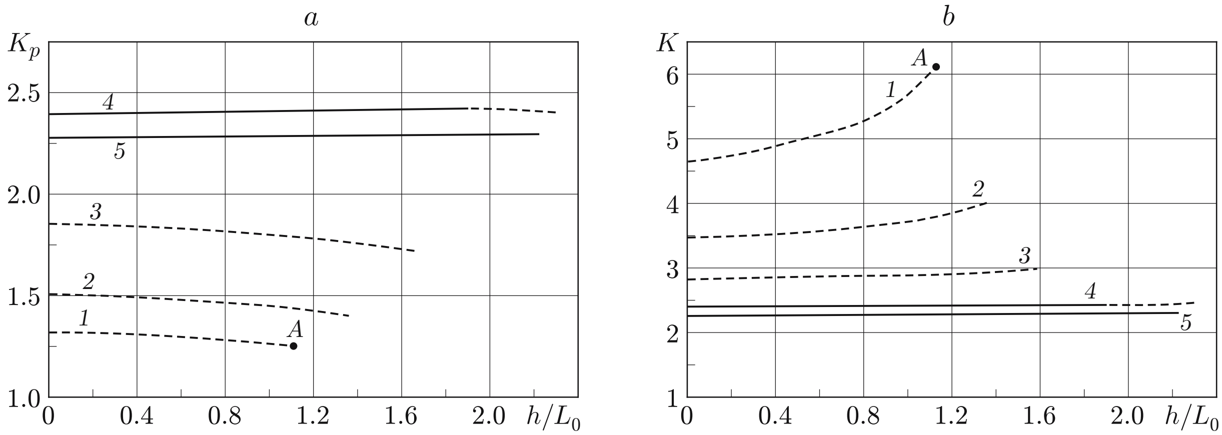

Fig. 5. Deformation coefficient K_p (a) and the relative penetration coefficient K (b) versus impactor penetration depth in sand: solid curves correspond to the hydrodynamic stage of deformation, and dashed curves to the plastic stage; $V_0 = 700$ (1), 850 (2), 1100 (3), 1750 (4), and 2500 m/sec (5); A is the point which corresponds to transition to the solid state.

determined by Lavrent'ev's formula taking into account the porosity of sand:

$$\frac{H}{L_0} = \sqrt{\frac{\rho_p}{C_0\rho_t}} = 1.87. \tag{5}$$

It should be noted that the wear depth determined by Lavrent'ev's theory differs greatly from the value obtained using the IMHT, which indicates an important role of the strength of the impactor in the examined velocity range during penetration into a low-strength target.

The results of calculations of the deformation coefficient $K_p = h/\Delta L_p$ and the relative penetration coefficient $K = h/\Delta L$ show that, for $V_0 > V_p$, K_p and K depend weakly on the penetration depth (Fig. 5). In the hydrodynamic stage of penetration, by virtue of above the assumption that $\Delta L_p = \Delta L$, we have the equality $K = K_p$.

Thus, the depth corresponding to the end of deformation can be calculated by the formula $H = K_pL_0$ (points in Fig. 4). The calculated depth agree well with the experimental values of H/L_0 and the difference is explained, in particular, by the fact that the values of H/L_0 were estimated from experimental values of K_p ignoring the nonstationarity of the process.

In addition, system (2) implies that, for $h \to 0$, the deformation coefficient K_p can be calculated from the following formulas:

— in the plastic stage,

$$K_{p0} = \frac{dh}{d(\Delta L_p)} = \frac{U_0}{C_p}; \tag{6}$$

— in the hydrodynamic stage

$$K_{p0} = K_0 = \frac{dh}{d(\Delta L)} = \frac{U_0}{V_0 - U_0} \tag{7}$$

[U_0 is determined from (1) taking into account the dependence $Y_p(V_0)$]. The calculations show that during impactor penetration, the difference between the values of K_p and K_{p0} does not exceed 5–8% in the plastic stage (region II in Fig. 5a) and 1% in the hydrodynamic stage (regions III and IV). In view of the above-mentioned differences, to determine the depth corresponding to the end of deformation in regions II, III, and IV (in these regions, the solid part of the impactor disappears), it is possible to use the approximate relation (see Fig. 4)

$$H/L_0 = K_{p0}(V_0). \tag{8}$$

In the region $V_p < V_0 < V_{ht}$, the length of the plastically compressed part of the rod (the so-called mushroom in Fig. 1a) is equal to

$$\Delta L_{mr} = h\Big(\frac{1}{K_{p0}} - \frac{1}{K_0}\Big) = \Delta L_p\Big(1 - \frac{K_{p0}}{K_0}\Big), \tag{9}$$

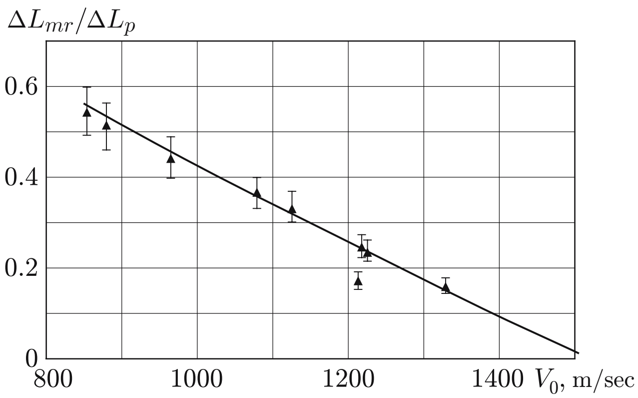

Fig. 6

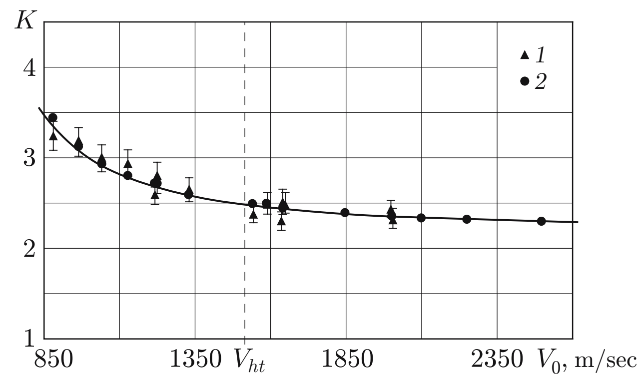

Fig. 7

Fig. 6. Relative length of the region of plastic deformation of the impactor versus penetration velocity: the solid curve corresponds to the calculation using the analytical dependence (9); points are experimental data.

Fig. 7. Relative penetration coefficient versus penetration velocity: points 1 refer to experimental data and points 2 to calculation using the IMHT; the solid curve corresponds to the calculation using the analytical dependence.

i.e., it is proportional to the penetration depth. In Fig. 6, the values of ΔL_{mr}, determined by dependence (9) are shown in comparison with experimental data (see Table 1). The dependence of the relative penetration coefficient K of the copper impactor on V_0 for $V_0 > V_p$ is presented in Fig. 7.

4. Conclusions. High-velocity penetration of copper impactors into sand was studied experimentally using x-ray photography, which made it possible to determine the velocity and characteristics of rod deformation.

An improved modified hydrodynamic theory was proposed to describe both the plastic and hydrodynamic stages of impactor penetration into a low-strength target. The effectiveness of the theory is confirmed by the results of comparison of calculated and experimental data.

Four ranges of penetration velocities of copper impactors into a low-strength porous (sandy) target are determined, in which the characteristics of impactor deformation and the depth corresponding to the end of deformation differ significantly.

It is shown that in the range of penetration velocities in which complete deformation of the rod occurs, the relative penetration and deformations coefficients depend weakly on penetration depth. This allows one to approximately estimate the depth corresponding to the end of deformation using the analytical relation (8).

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